(Fractional) Online Stochastic Matching via Fine-Grained Offline Statistics

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Introduction

Our framework

Algorithms / Analysis

Online Stochastic Matching

[Feldman, Mehta, Mirrokni, and Muthukrishnan, 2009]

INPUT Bipartite graph $G = (L \cup R, E)$

- Vertices $u \in L$ are **offline**.
- Vertices $j \in R$ are **online**.

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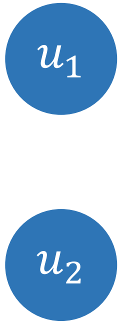

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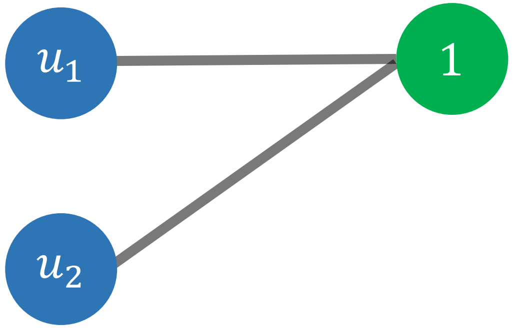

$t_1 = \{u_1, u_2\}$

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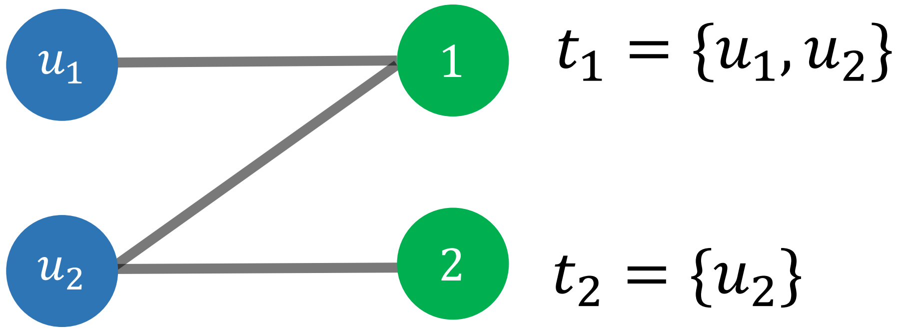

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OUTPUT Algorithm must decide irrevocably matching for $j \in R$.

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Goal

- Unweighted: Maximize the cardinality of the matching
- Vertex Weighted: Maximize the total weight of matched $\mathrm{u} \in L$.

Competitive Ratio

Metric The ratio between algorithm and offline optimum.

$$\mu = \min_{G, D_1, D_2, \cdots, D_n} \frac{\mathrm{E}\,[\,\mathrm{ALG}(G)\,]}{\mathrm{E}\,[\,\mathrm{OPT}(G)\,]}$$

Previous Works

- **IID arrival:** Type distributions D_j are the same for all $j \in R$.

Arrival	Goal	Ratio	
IID	Unweighted	0.711	[Huang and Shu, 2021]
	Vertex-weighted	0.701	
Non-IID	Vertex-weighted	$1 - 1/e \approx 0.632$	[Aggarwal, Goel, Karande, and Mehta, 2011]

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Parallel to our work, Huang, Shu, and Yan improved the ratio for IID vertex-weighted setting to 0.716 .

Key Idea

- **Warm up:** One – Choices Algorithm
  - When j arrives, we sample **one** neighbor of it.
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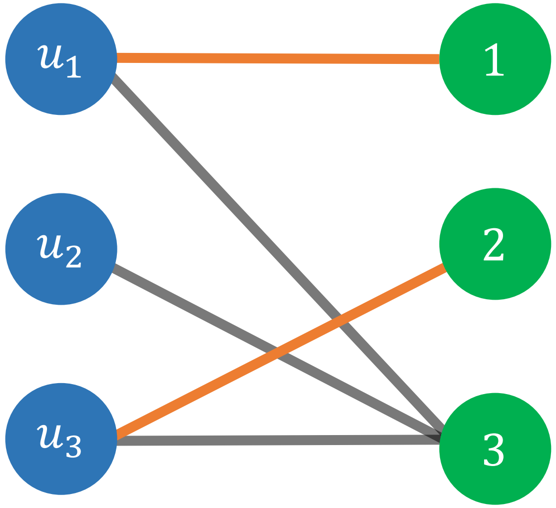
    it is sampled with probability $\Pr[\,(u,j) \in \mathrm{OPT} \mid t_j\,]$.



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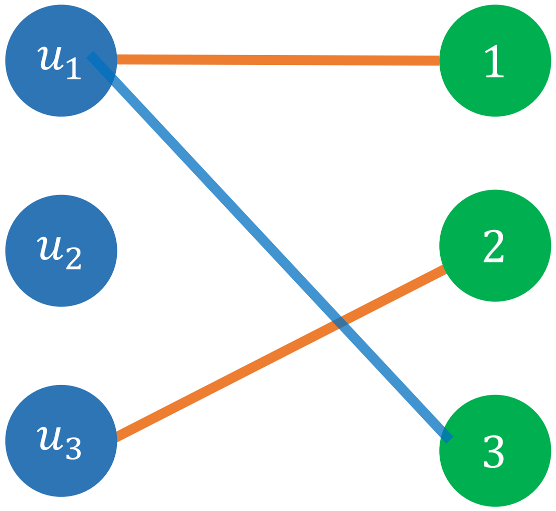

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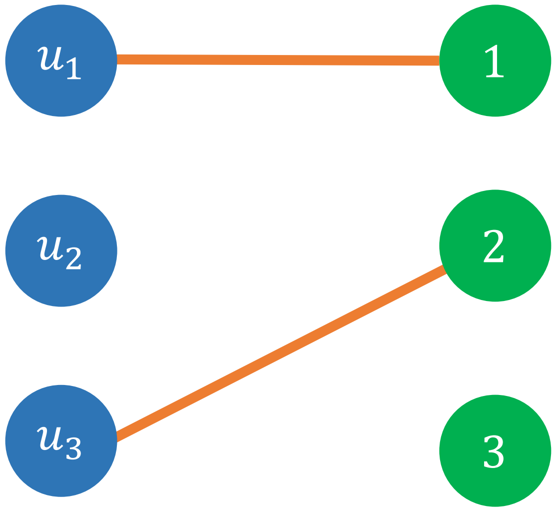

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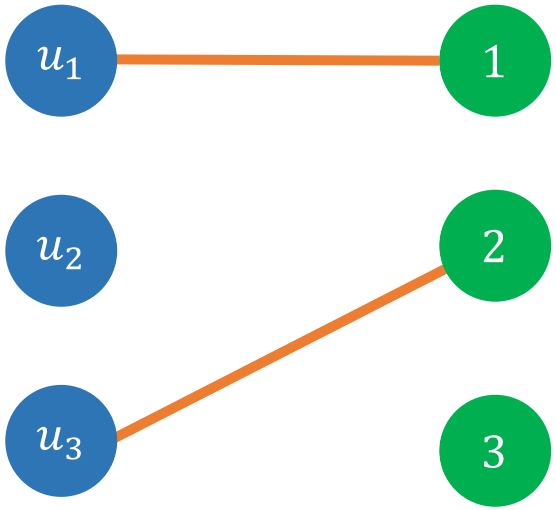

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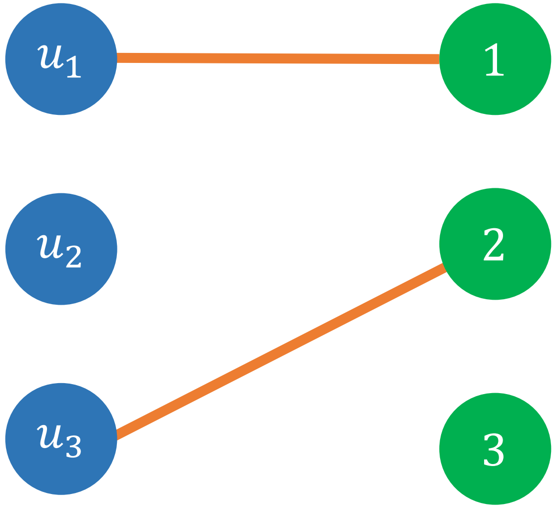

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  - $\Pr[\,u \in \text{ALG}\,] = 1 - \prod_j (1 - p_{u,j}) \geq 1 - e^{-p_u} \geq \left(1 - \frac{1}{e}\right) p_u$

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$$
\begin{aligned}
\mathrm{E}[\,\mathrm{ALG}\,] &\geq \sum_u w_u \cdot \Pr[u \in ALG] \\
&\geq \left(1 - \frac{1}{e}\right) \sum_u w_u p_u \\
&= \left(1 - \frac{1}{e}\right) \mathrm{E}[\mathrm{OPT}]
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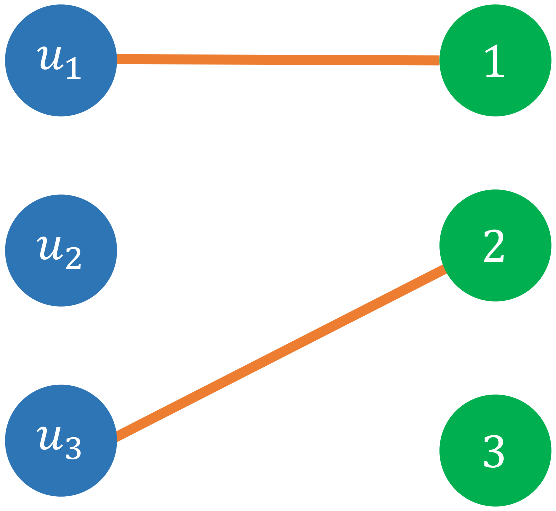

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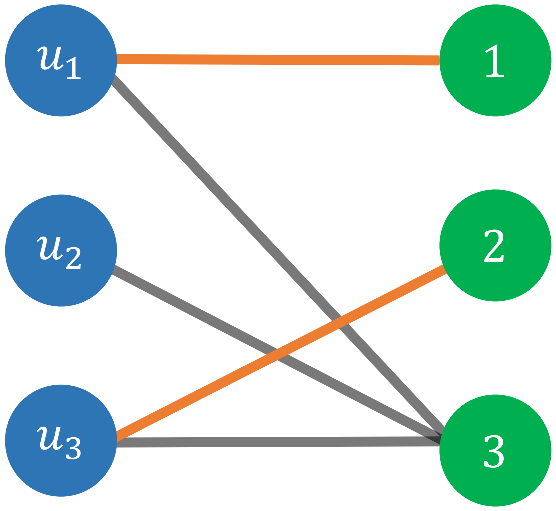

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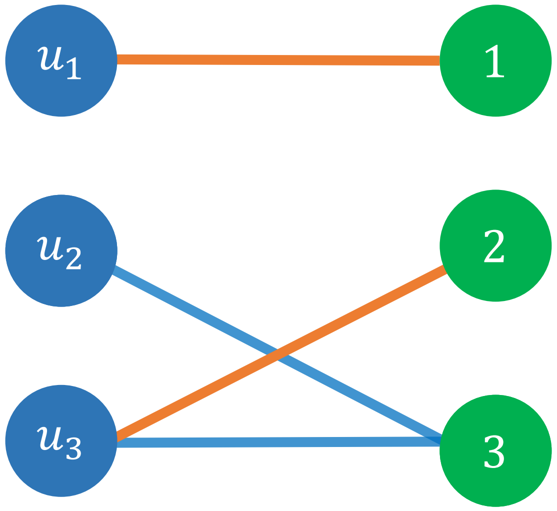

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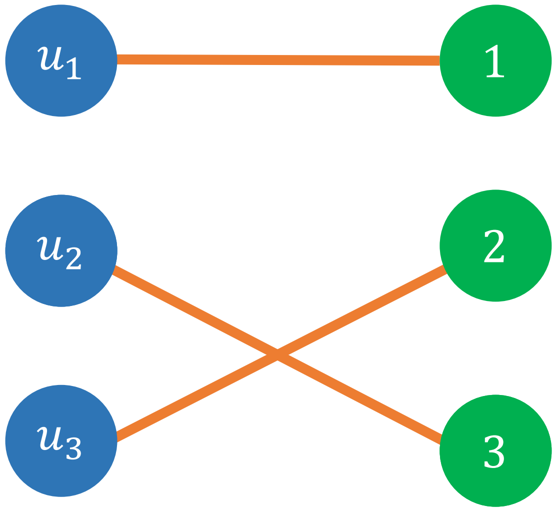

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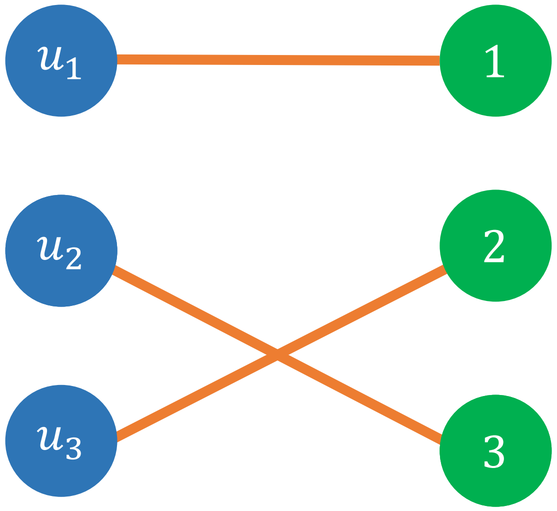

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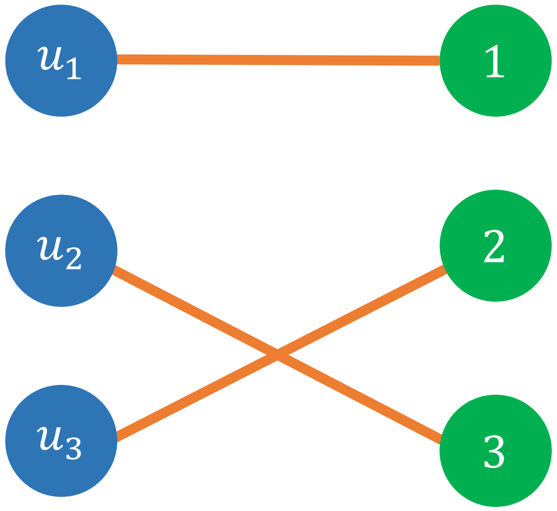

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Parallel to our work, Huang, Shu, and Yan also exploit the power of multi-selection.

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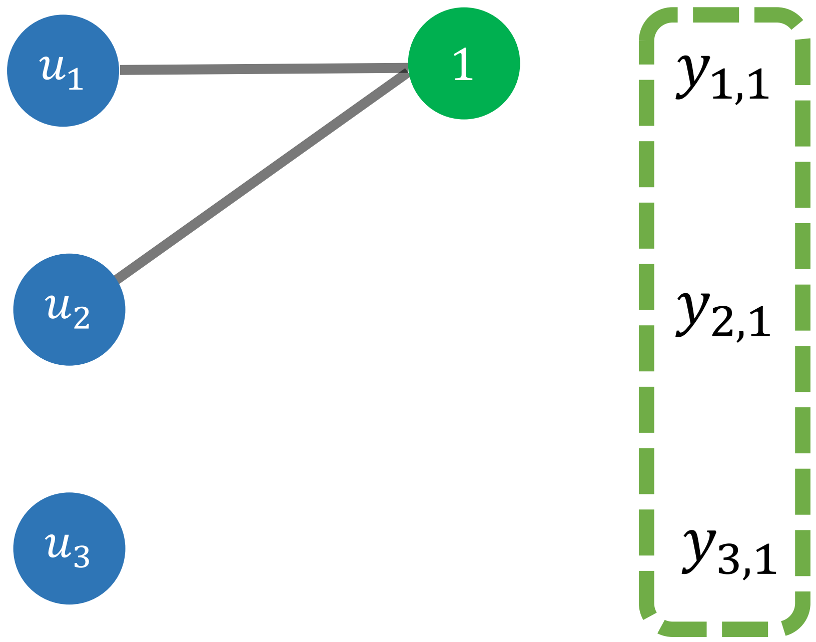
Upon the arrival of each $j \in R$, the algorithm matches it with each $u \in L$ with fraction $y_{u,j}$.



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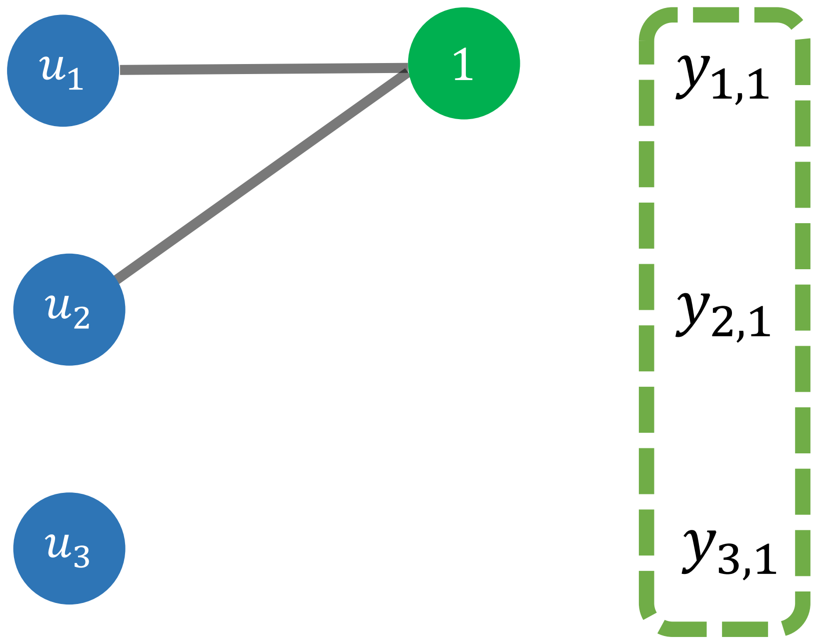

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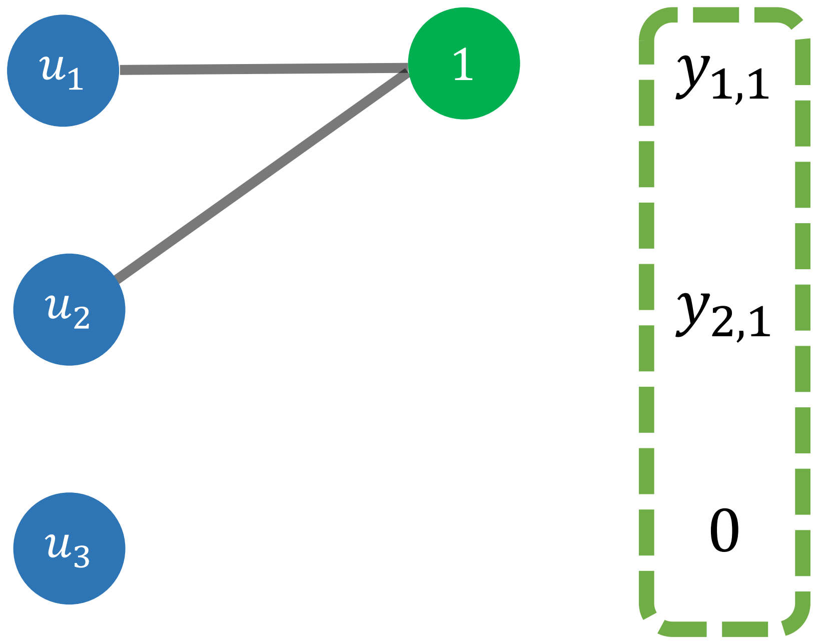

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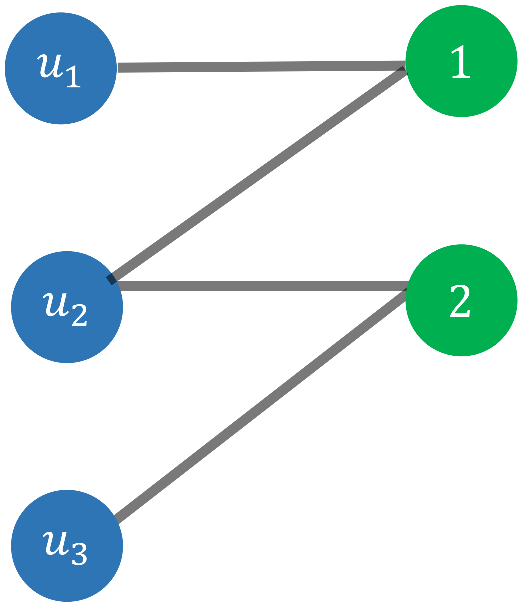

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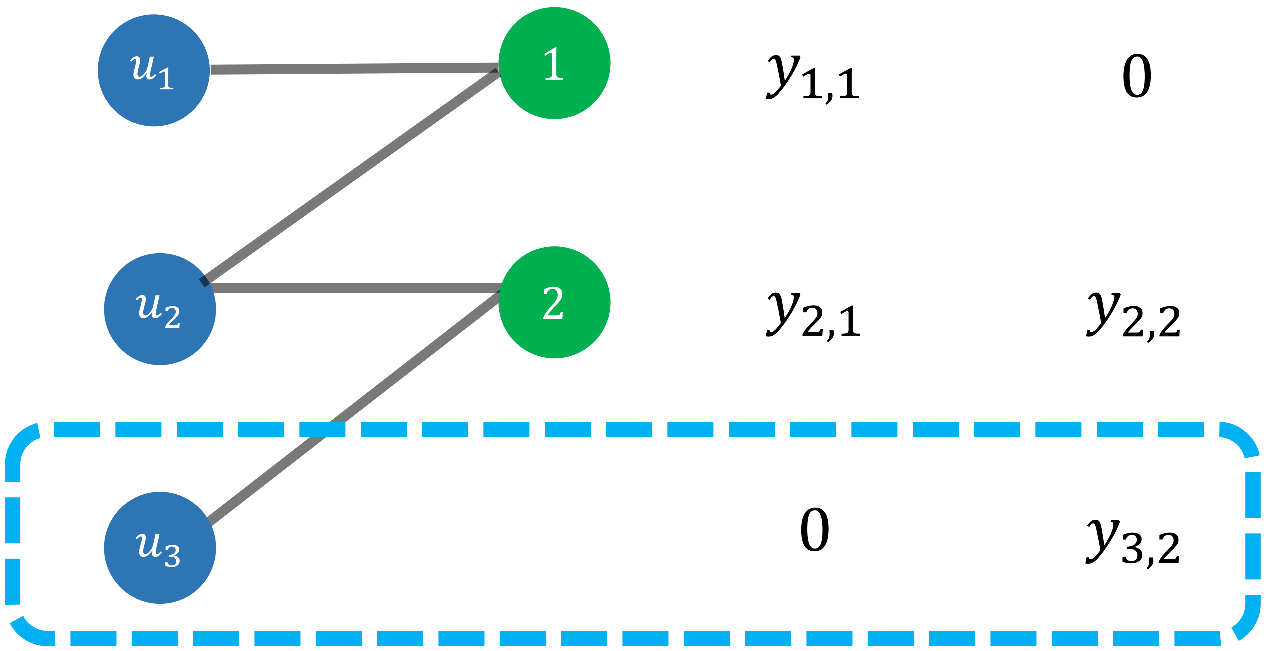

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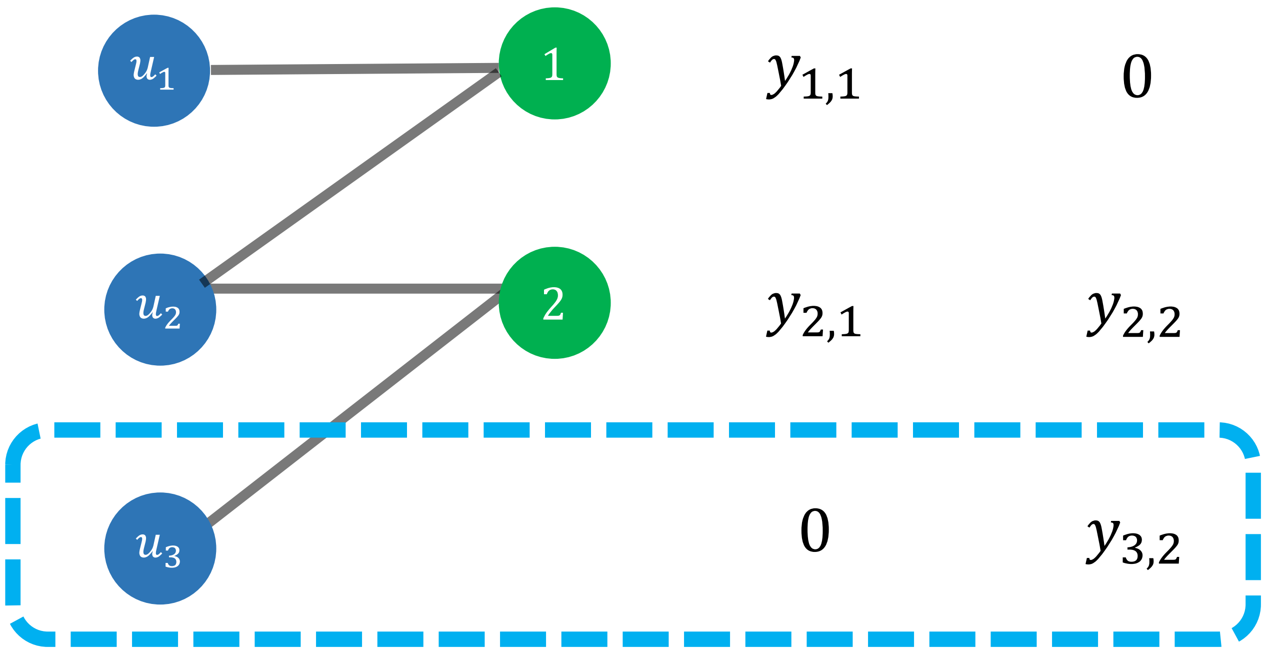

- We define $y_u := \sum_j y_{u,j}$.
- The weight of fractional matching is defined as

$$\mathrm{FRAC} := \sum_u w_u \cdot \min(y_u, 1)$$

Independent Rounding

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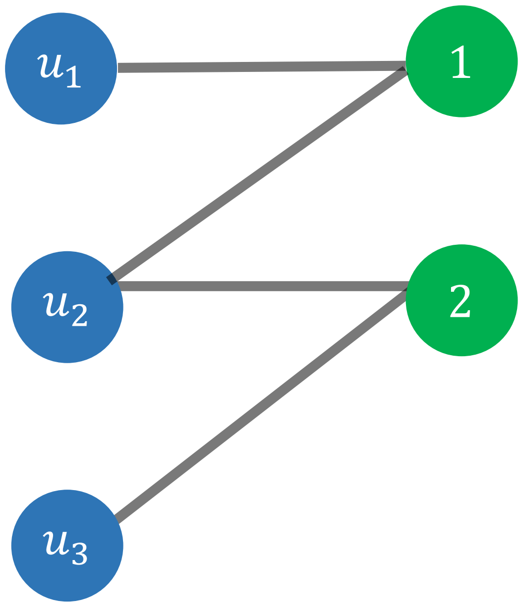

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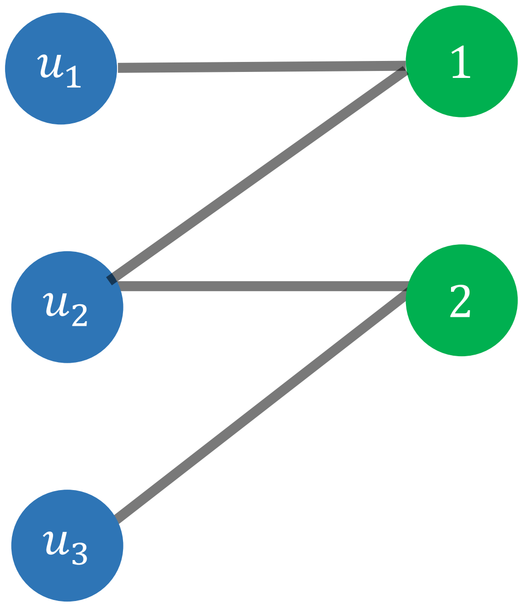

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- The weight of matching afterward is

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$$= \sum_u w_u \cdot (1 - e^{-y_u})$$

Rounding with Online Correlated Selection

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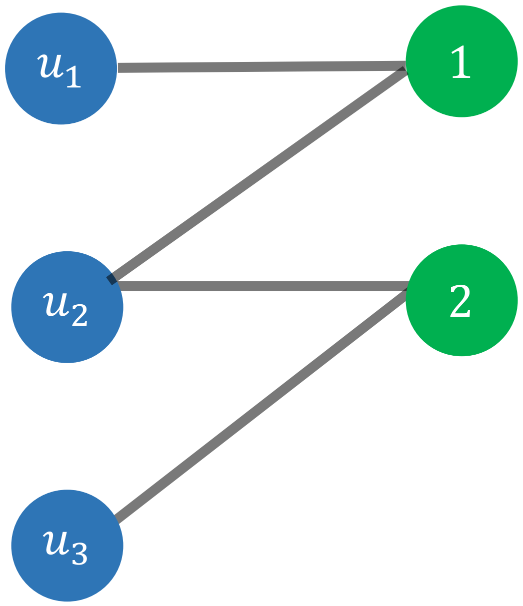
For each $j \in R$, we apply OCS with y_j as input.



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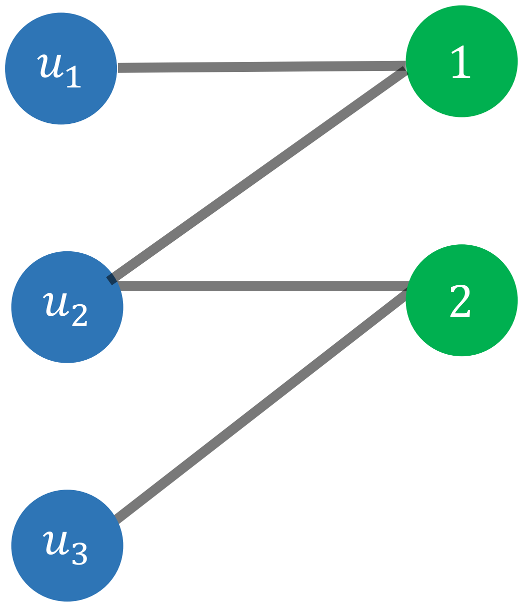

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Our Framework

Lemma.

Let $y_u = \sum_j y_{u,j}$. The algorithm would achieve performance

$$\sum_u w_u \cdot f(y_u)$$

where

$$f = \begin{cases} \min(1, y_u) & \text{for fractional matching} \\ & \end{cases}$$

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- $y_{u,j} = 0$ if there is no such edge.
- $y_u = \sum_j y_{u,j}$ has $\mathrm{E}[y_u] = \Pr[\, u \in \mathrm{OPT} \,]$

Main Difficulty

Lemma.

Any **unbiased estimators** $y_{\mathrm{u,j}}$ with $\mathrm{E}[\, f(y_u)\,] \geq \mu \cdot \mathrm{E}[\, y_u \,]$ implies a μ-competitive algorithm.

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Proof

$$\mathrm{E}[\,\mathrm{ALG}\,] = \mathrm{E}\left[\sum_u f(y_u) \cdot w_u\right] \geq \mu \cdot \sum_u E[y_u] \cdot w_u$$

$$= \mu \cdot \sum_u \Pr[\, u \in OPT \,] \cdot w_u \;= \mu \cdot \mathrm{E}[\,\mathrm{OPT}\,]$$

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But y_u is a **random variable**.

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Any **unbiased estimators** $y_{\mathrm{u,j}}$ with $\mathrm{E}[\, f(y_u)\,] \geq \mu \cdot \mathrm{E}[\, y_u \,]$ implies a μ-competitive algorithm.

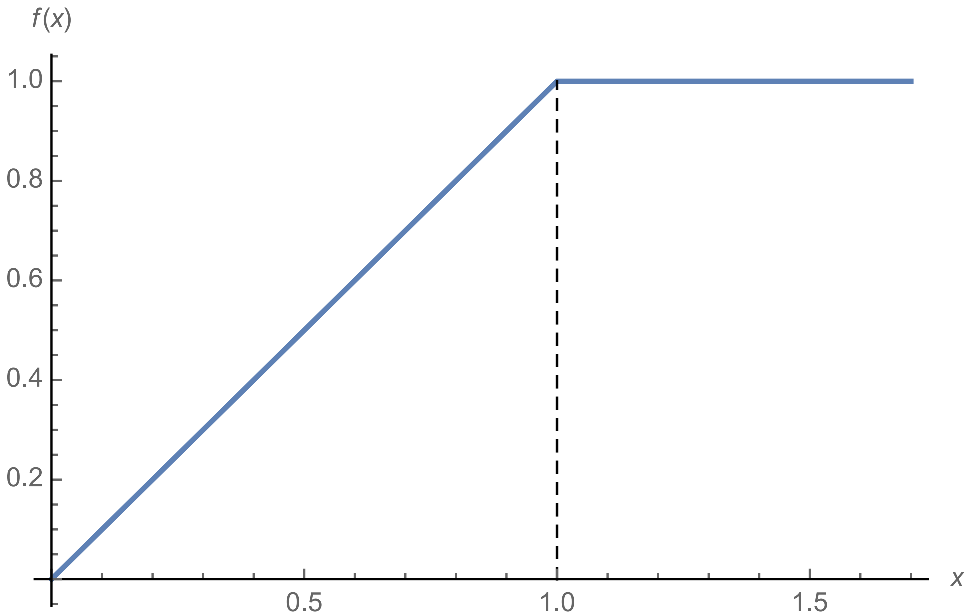
Main Difficulty.

For example, let $f(y) = \min(1, y)$.

We know $\mathrm{E}[\, y_u \,] = \Pr[\, u \in \mathrm{OPT} \,] \in [0,1]$.
For any **deterministic** y ∈ $[0,1]$, we do have $f(y) = y$.
But y_u is a **random variable**. It can be larger than 1!

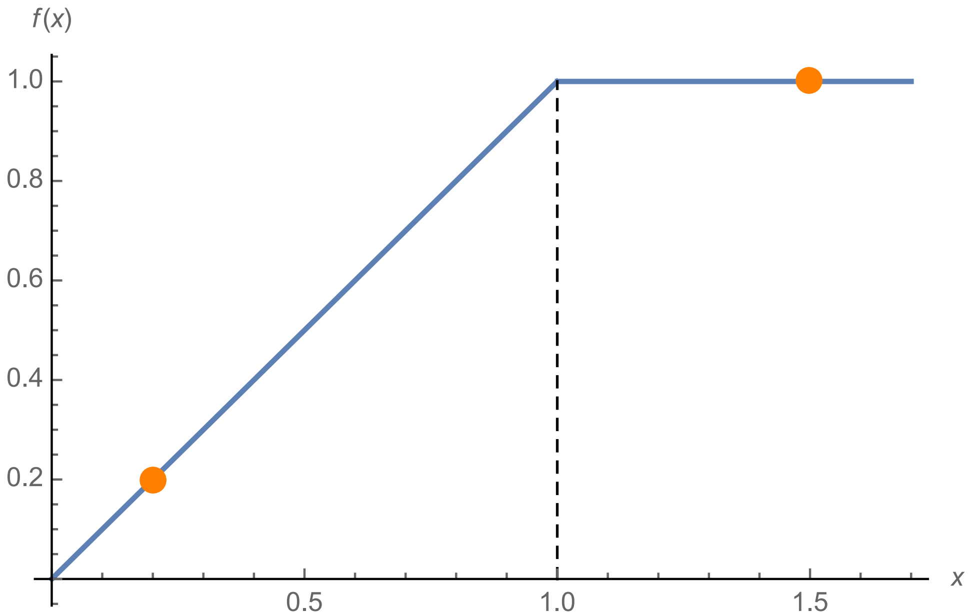
Main Difficulty

Function $f(y)$ is **concave**.



Main Difficulty

Function $f(y)$ is **concave**.
Suppose y_u is a even mixing of 0.2 and 1.5.



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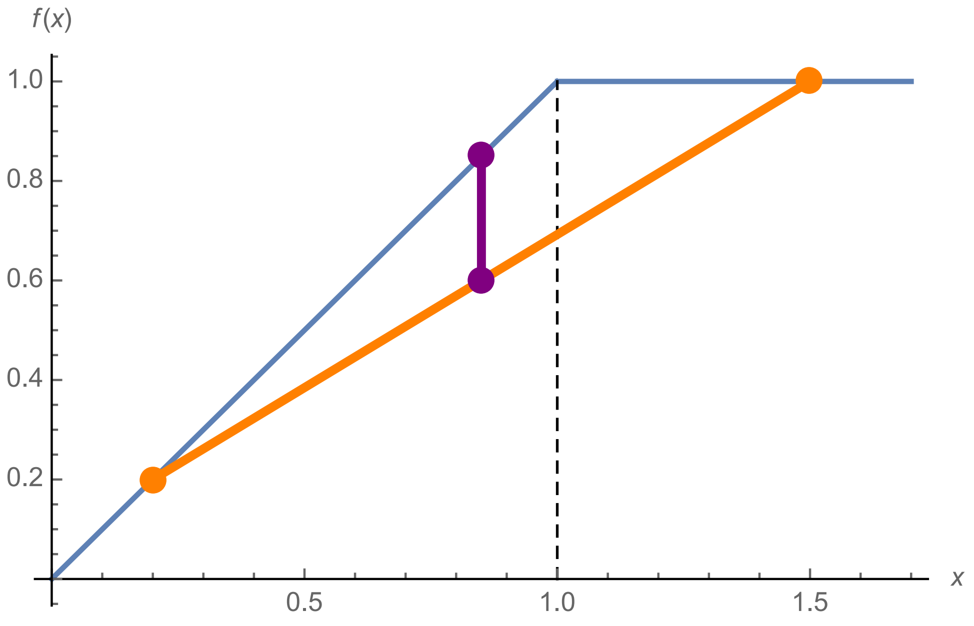
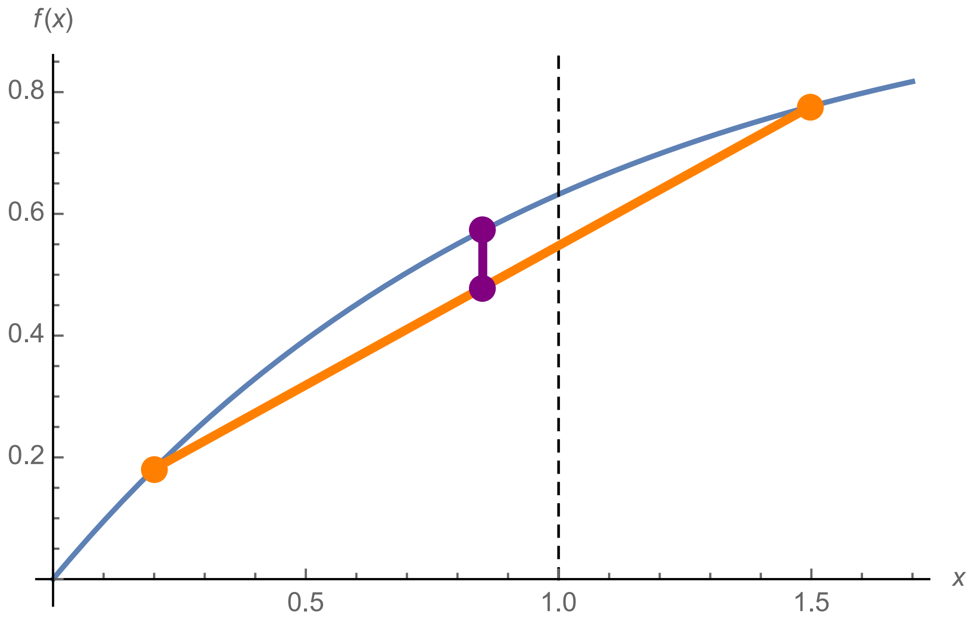
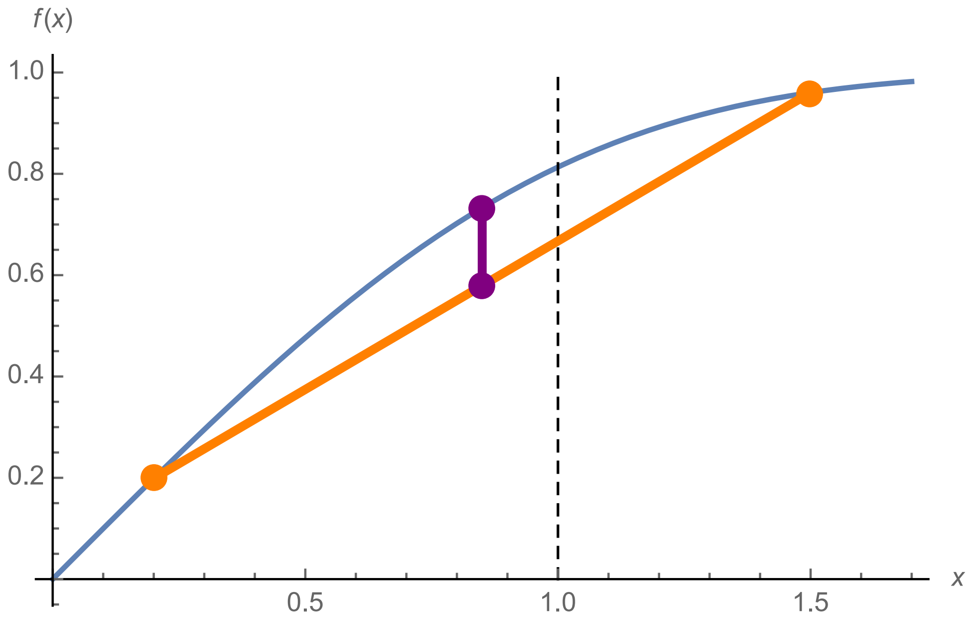


$$\mathrm{E}[\, f(y_u)\,] \ll \mathrm{E}[\, y_u\,]$$

Main Difficulty

Same issue for other two functions.

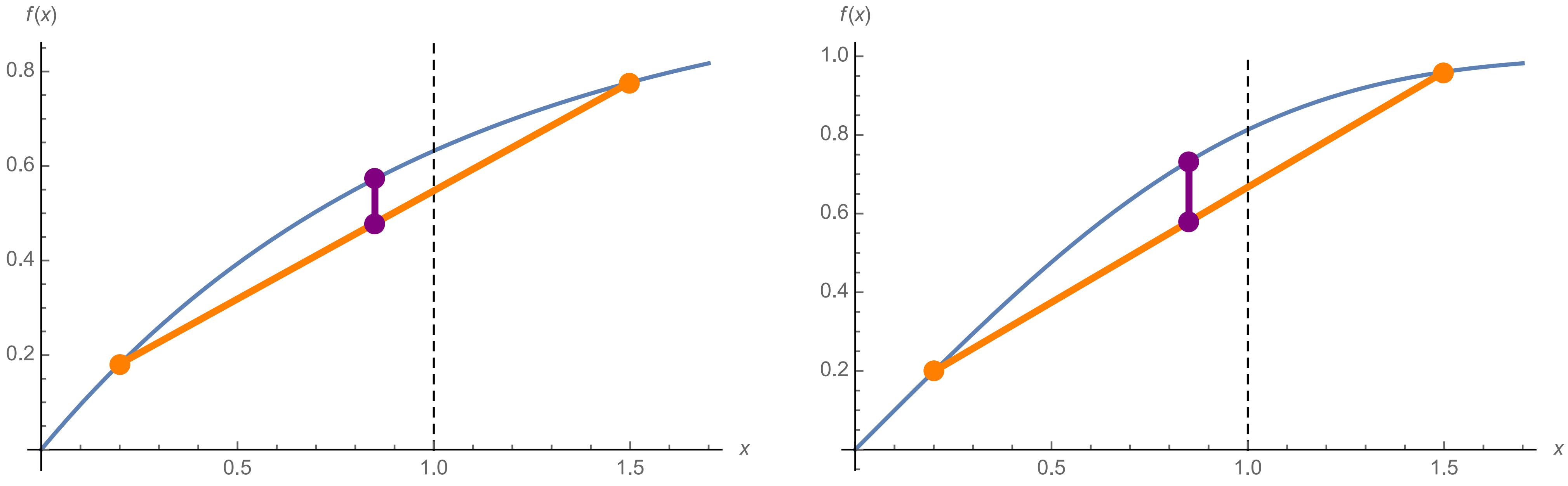




$$\mathrm{E}[\, f(y_u)\,] \ll f(\mathrm{E}[\, y_u\,]) \leq \mathrm{E}[y_u]$$

Main Difficulty

Same issue for other two functions.



We must bound the **spread** of y_u !

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IID arrival

Non-IID arrival

IID Arrival: Bounding Variance

Bound the **spread** of a random variable

→

Bound the **variance** of a random variable

IID Arrival: Bounding Variance

Bound the **spread** of a random variable

Bound the **variance** of a random variable

Lemma.

For any random variable y_{u} with variance σ and any (concave) function f, there exists a constant $\mu(\sigma, f)$ satisfying $\mathrm{E}[\, f(y_u)\,] \geq \mu(\sigma, f) \cdot \mathrm{E}[\, y_u \,]$

IID Arrival: Tradeoff

Our Goal.

Design Unbiased Estimators $y_u = \sum_j y_{u,j}$ with minimal variance.

IID Arrival: Estimators

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Design Unbiased Estimators $y_u = \sum_j y_{u,j}$ with minimal variance.

Independent Estimators. $\mathrm{y}_{u,j}(t_j) = \Pr[(u,j) \in \mathrm{OPT} \mid t_j]$

IID Arrival: Estimators

Our Goal.
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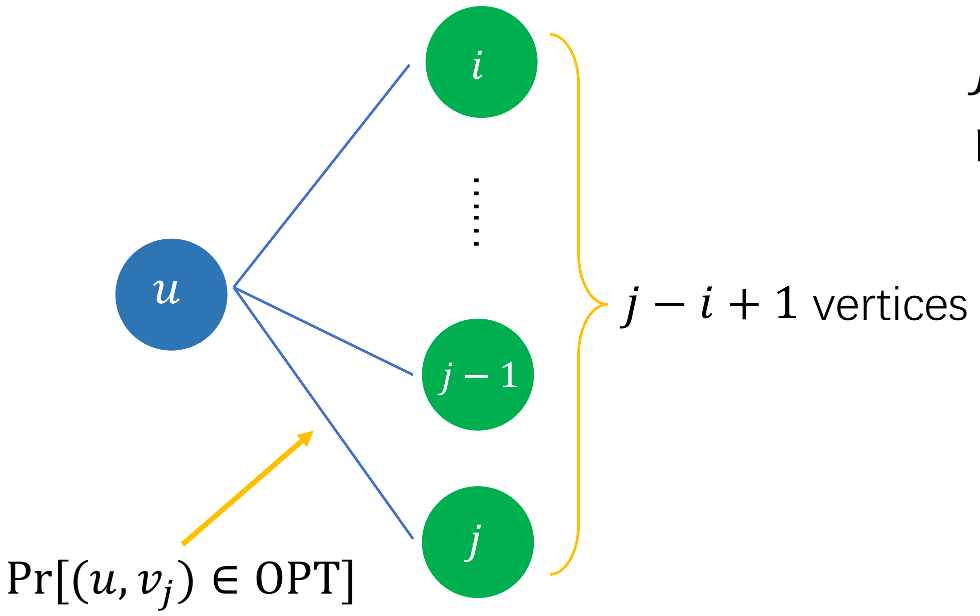
Independent Estimators. $\mathrm{y}_{u,j}(t_j) = \Pr[(u,j) \in \mathrm{OPT} \mid t_j]$

Fully-Correlated Estimators. $\mathrm{y}_{u,j}\left(t_1, t_2, \ldots, t_j\right) = \Pr[(u,j) \in \mathrm{OPT} \mid t_1, t_2, \ldots, t_j]$

IID Arrival: Windowed Estimators



Windowed Estimators. Fix the types of the last $j - i + 1$ arrived vertices and resample the remaining types.

$$y_{u,j}^{[i]}(t_i, t_{i+1}, \dots, t_j) = \Pr[(u, j) \in \mathrm{OPT} \mid t_i, t_{i+1}, \dots, t_j]$$

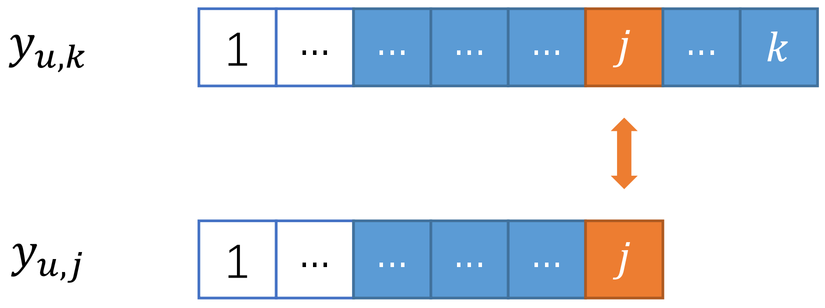
IID Arrival: Tradeoff

$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\leq j,k}}\left[y_{u,j} \cdot y_{u,k}\right]$$

IID Arrival: Tradeoff

$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\le j,k}}\left[y_{u,j} \cdot y_{u,k}\right]$$

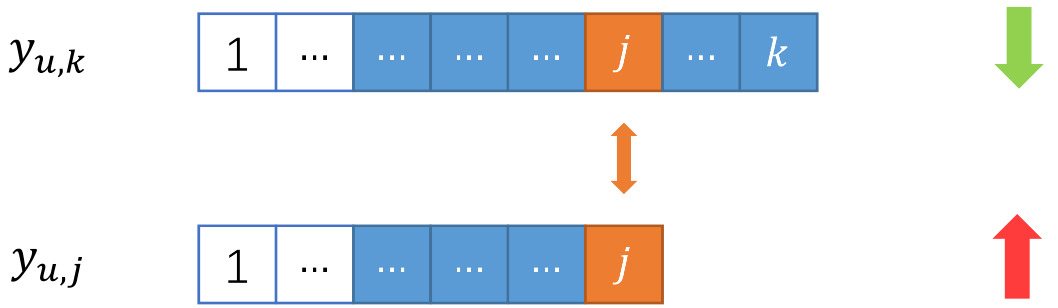
$y_{u,k}$ conditions on t_j



IID Arrival: Tradeoff

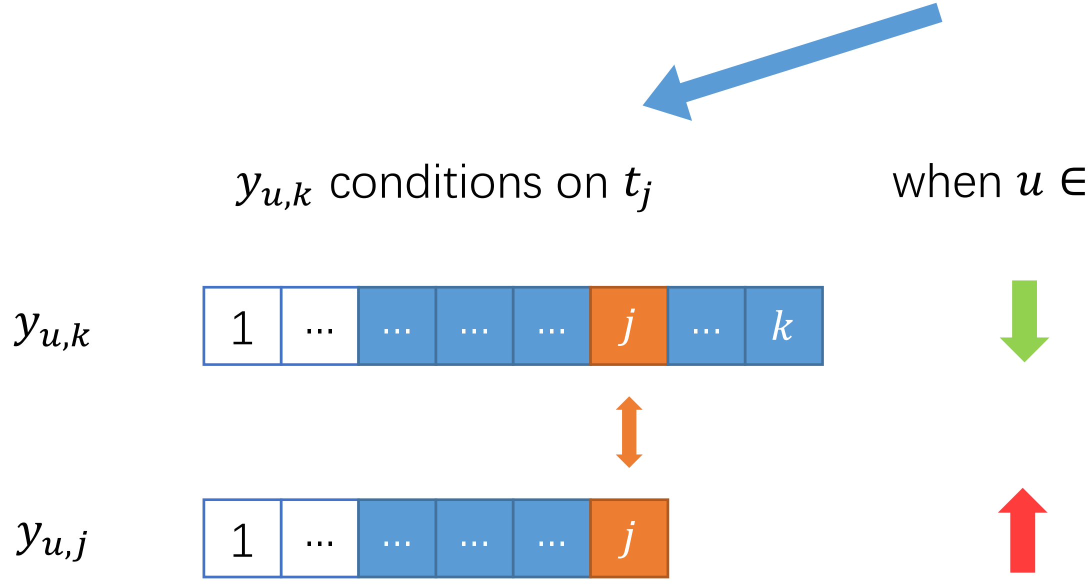
$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\leq j,k}}[y_{u,j} \cdot y_{u,k}]$$

$y_{u,k}$ conditions on t_j when $u \in t_j$



IID Arrival: Tradeoff

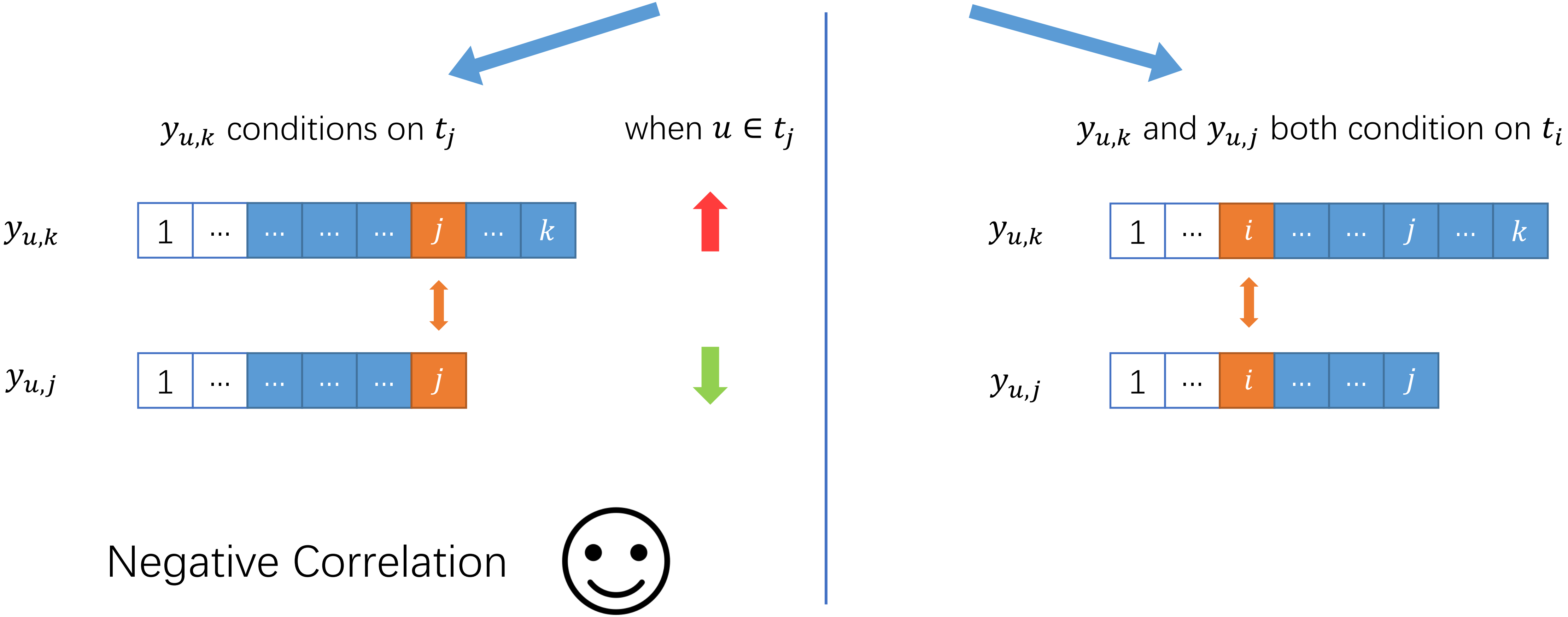
$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\leq j,k}}[y_{u,j} \cdot y_{u,k}]$$



Negative Correlation

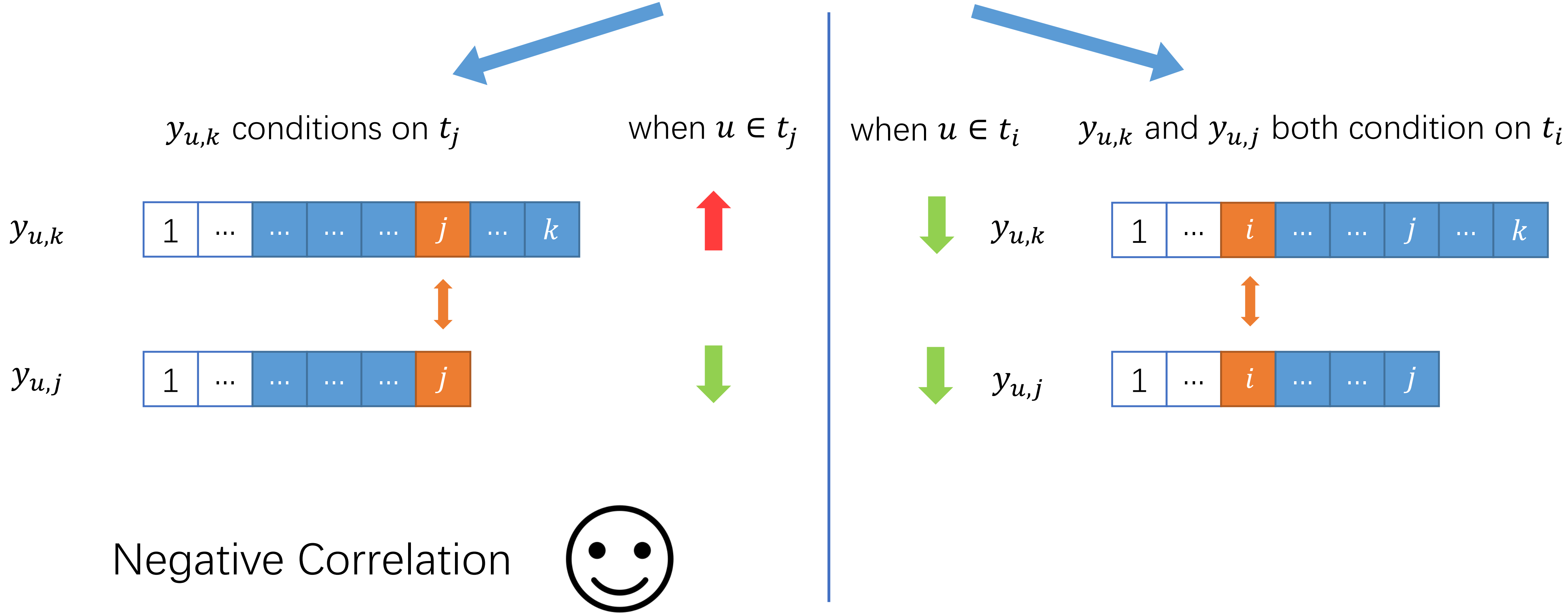
IID Arrival: Tradeoff

$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\le j,k}}[y_{u,j} \cdot y_{u,k}]$$



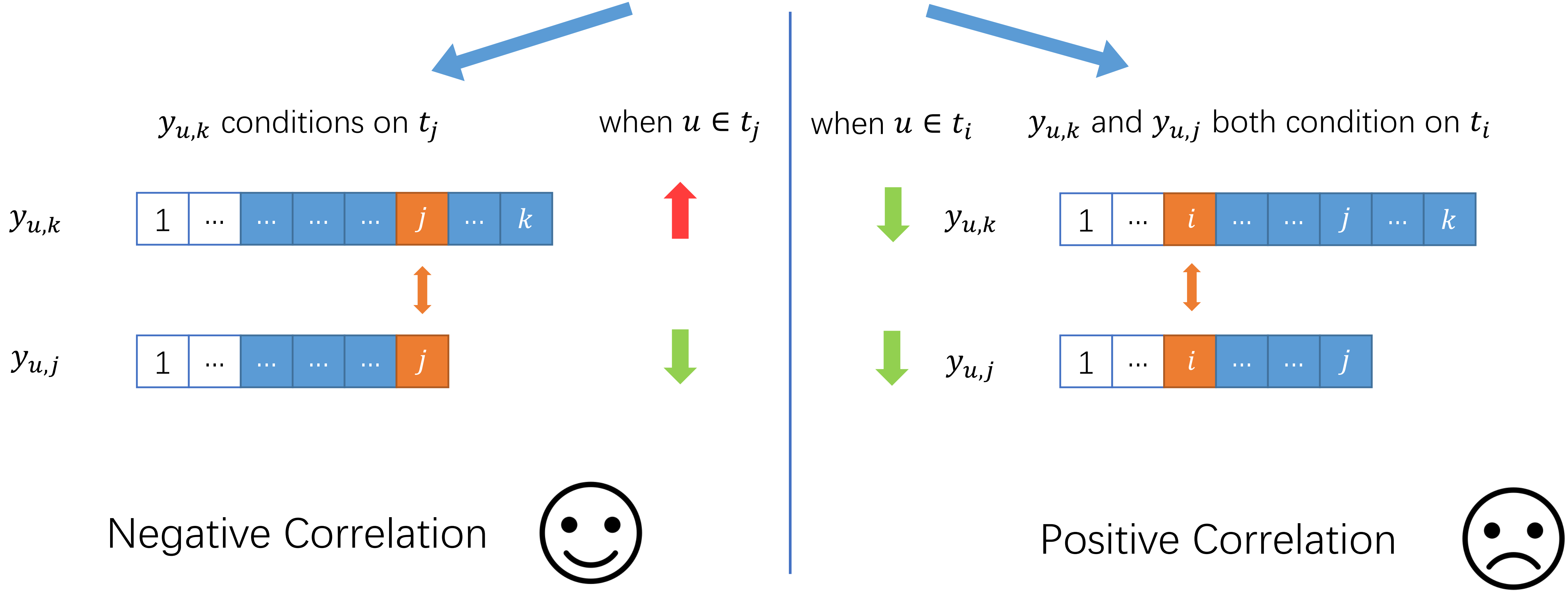
IID Arrival: Tradeoff

$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\leq j,k}}[y_{u,j} \cdot y_{u,k}]$$

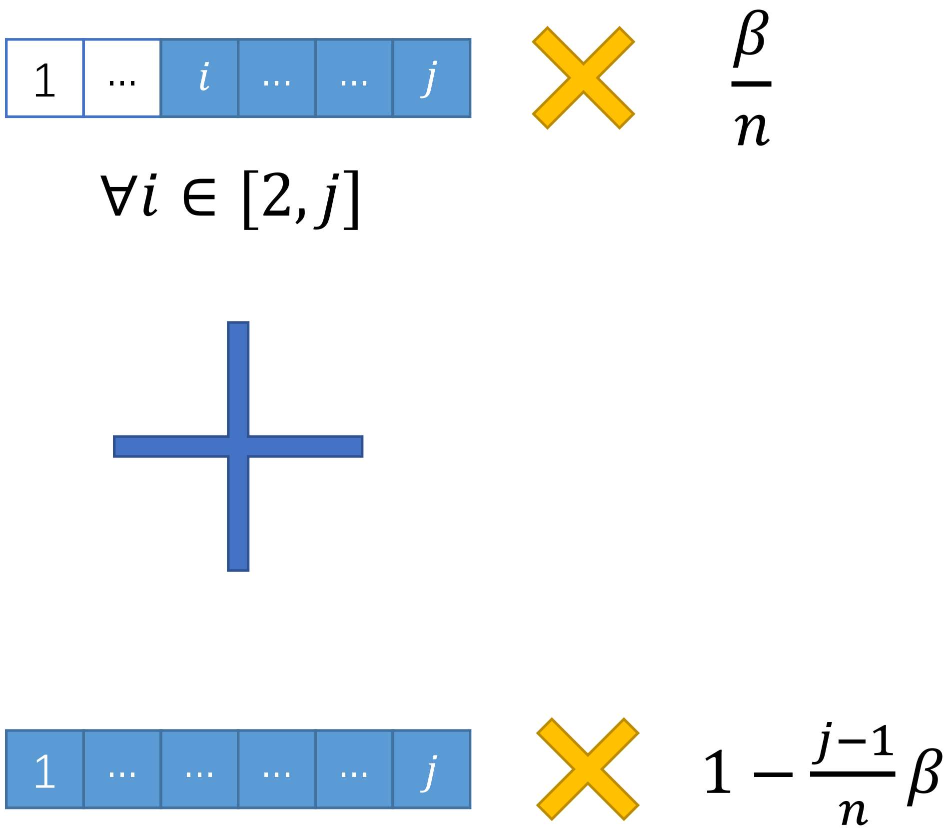


IID Arrival: Tradeoff

$$\mathrm{E}_{\mathbf{t}}[y_u^2] = \sum_{j,k} \mathrm{E}_{\mathbf{t}_{\leq j,k}} [y_{u,j} \cdot y_{u,k}]$$



IID Arrival: Mixing of Windowed Estimators



Our Estimators. The estimators we used is a linear combination of windowed estimators:

$$y_{u,j} = \frac{\beta}{n}\sum_{i=2}^{j} y_{u,j}^{[i]} + \left(1 - \frac{j-1}{n}\beta\right) y_{u,j}^{[1]}$$

where $\beta = 0.79$ is a optimized constant.

Introduction

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Non-IID: Independent Estimator is Optimal

Independent Estimators. $y_{u,j} = \Pr[(u,j) \in \mathrm{OPT} \mid t_j]$.

Proof Sketch.

1. For any fixed mean $\mathrm{E}[\, y_u \,]$, we characterize the **worst-case** distribution that minimizes $\mathrm{E}[\, f(y_u) \,]$.

2. Any unbiased estimator has the same performance under the worst-case distribution.

Thanks!